A model of peak production in oil fields

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We developed a model for oil production on the basis of simple physical considerations. The model provides a basic understanding of Hubbert's empirical observation that the production rate for an oil-producing region reaches its maximum when approximately half the recoverable oil has been produced. According to the model, the oil production rate at a large field must peak before drilling peaks. We use the model to investigate the effects of several drilling strategies on oil production. Despite the model's simplicity, predictions for the timing and magnitude of peak production match data on oil production from major oil fields throughout the world.

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I. INTRODUCTION

The possibility that worldwide production of crude oil may soon peak has received considerable attention.[1–10] Numerous studies of global supply and demand have projected the production rate to peak within the next several decades, although significant controversy remains as to the likely timing.[3–10] Most studies rely on the empirical observation that the production rate for an oil-producing region reaches its maximum near the midpoint of production, that is, when approximately half the recoverable oil has been produced. This "decline-from-the-midpoint" approach was most famously used by Hubbert in his successful prediction of peak U.S. oil production.[1]

In this paper we develop a model of oil production that provides insight into the physical basis for the decline-from-the-midpoint behavior and matches historical data from major oil fields around the world. The model relates the number of producing wells to future production, providing a straightforward method for estimating peak output at working fields. The model can also be used to investigate different strategies for developing oil fields. We show analytically that the production rate at any given field must peak before the number of active wells peaks.

After a reservoir is located, oil field development usually begins with primary recovery in which oil is allowed to flow out of new wells under its own pressure, with nothing reinjected into the reservoir. During primary recovery, downhole pressure (the fluid pressure at the entry to a well pipe) drops as oil is produced. This gradual reduction in pressure limits the production rate of oil and effectively restricts the recoverable oil to a fraction of the total reservoir.

In oil fields of significant size, secondary recovery ordinarily overshadows primary recovery.[11,12] In secondary recovery, fluid (for example, water) is reinjected into a field as oil is removed, thereby maintaining nearly constant downhole pressure. This procedure allows a greater fraction of the reservoir to be recovered and provides for more control over the production rate.

We will focus on a simple model of secondary oil recovery.[13] This simple model is not intended to capture the effects of enhanced oil recovery techniques that are typically used near the end of an oil field's lifetime, well after peak production.

II. THE MODEL

We treat an oil reservoir as a sealed container filled with incompressible crude oil (liquid petroleum) and other fluids at a downhole pressure P_{dh} much greater than atmospheric pressure P_{atm}. For simplicity, we assume that a single pipe, with a cross-sectional area $A(t)$ and a small volume compared to the volume of oil, extends into the container and represents the total area and volume of all active wells. Also for simplicity, we assume that $A(t)$ is a continuous function of time, the downhole fluids are well mixed, and the downhole fluid pressure and volume remain nearly constant due to reinjection.

Conservation of mass requires that the oil production rate (the volume of oil per unit time exiting the pipe) satisfies

$$\dot{V}(t) = u(t)A(t)x(t), \tag{1}$$

where $V(t)$ is the total volume of oil that has exited the well between time=0 and time=t, $u(t)$ is the velocity of the fluid mixture as it leaves the wellhead at time t, and $x(t)$ denotes the fraction of the output that is oil (rather than water or other impurities). A dot indicates differentiation with respect to time. Wellhead pressure (the fluid pressure at the exit to a well pipe) $P_{\mathrm{wh}}=P_{\mathrm{dh}}-\rho g h-P_{\mathrm{pf}}$, where h is the height of the column of fluid in the pipe, ρ is the density of fluid, and P_{pf} is the pressure drop due to steady-state pipe flow (either laminar or turbulent). The velocity of the fluid exiting the pipe can be derived from Bernoulli's principle (assuming incompressible oil at the wellhead) as

$$u(t)^2 = \frac{2(P_{\mathrm{wh}} - P_{\mathrm{atm}})}{\rho} \approx \frac{2P_{\mathrm{wh}}}{\rho}. \tag{2}$$

Thus, in the model, wellhead pressure exclusively determines the exit velocity. Constant downhole volume implies that

$$x(t) = x_0(1 - V/V_{\mathrm{rec}}), \tag{3}$$

where x_0 is the initial oil fraction and V_{rec} is the total volume of recoverable oil. The substitution of Eqs. (2) and (3) for $u(t)$ and $x(t)$ into Eq. (1) yields

$$\dot{V} = x_0\sqrt{\frac{2P_{\mathrm{wh}}}{\rho}}A\left(1 - \frac{V}{V_{\mathrm{rec}}}\right). \tag{4}$$

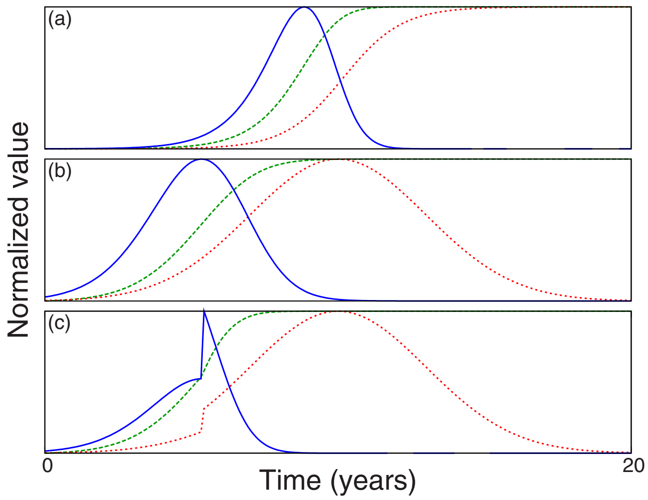

Fig. 1. The behavior of the model with various simple functional forms of $A(t)$. (a) Logistic $A(t)$ but not chosen in proportion to $V(t)$. (b) Gaussian $A(t)$. (c) Gaussian $A(t)$ suddenly doubled at the production rate peak. The dotted line indicates $A(t)$, the dashed line indicates the cumulative production $V(t)$, and the solid line indicates the production rate $\dot{V}(t)$.

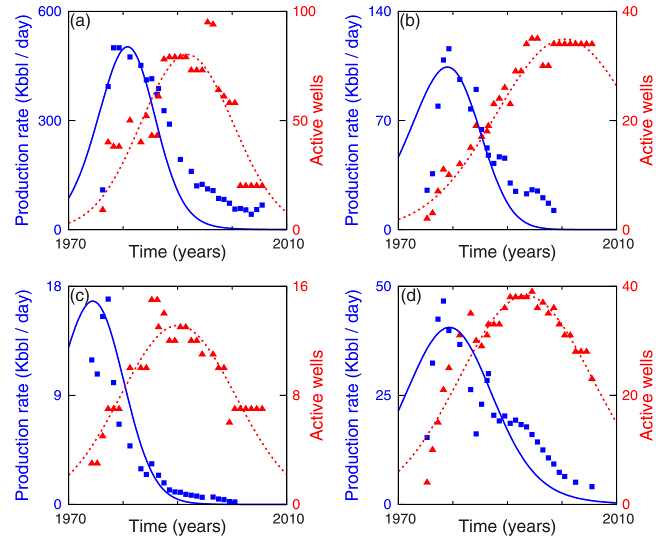

Fig. 2. Comparison of the model to oil fields on four continents. (a) Forties field, United Kingdom. (b) Ramadan field, Egypt. (c) Jaya field, Indonesia. (d) Poui field, Trinidad and Tobago. Triangles and squares are data for the number of active wells and oil production rate, respectively. Dotted curves are Gaussian best fits, and solid curves are corresponding solutions to Eq. (4). The parameters $(V_{\rm rec}, x_0)$ are as follows: (a) $(400\times 10^6\ \mathrm{m}^3, 0.36)$, (b) $(100\times 10^6\ \mathrm{m}^3, 0.35)$, (c) $(15\times 10^6\ \mathrm{m}^3, 0.10)$, and (d) $(50\times 10^6\ \mathrm{m}^3, 0.05)$; $P_{\rm wh}$ was fixed at 2 MPa. All parameters for the Forties field were determined from available data and not fitted (Refs. 16 and 17).

Equation (4) constitutes a simple model of secondary oil recovery in which three input parameters, the initial oil fraction, the wellhead pressure, and the total volume of recoverable oil, must be specified.[15] These parameters are often available or can be estimated from publicly accessible data for working oil fields.

III. MODEL PREDICTIONS

In a real oil field the cross-sectional area of active wells, which varies as wells are opened or shut down, determines production. Similarly, in the model we must specify the time-varying area $A(t)$ to solve Eq. (4) for the oil production $V(t)$ and its rate $\dot{V}(t)$. By specifying various functions for $A(t)$, the model can be used to explore the effect of different drilling strategies on production. Figure 1 shows three functional forms for $A(t)$ and the resulting solutions of Eq. (4) for $V(t)$ and $\dot{V}(t)$. In each case $A(t)$ has the same peak value, and realistic values for x_0, $P_{\rm wh}$, and $V_{\rm rec}$ were used. Figure 1(a) displays a sigmoid-shaped function where $A(t)$ initially grows rapidly and then slows down and levels off. The peak in $\dot{V}(t)$ occurs when 59% of the recoverable oil has been extracted. In Fig. 1(b) a Gaussian is used for $A(t)$, and $\dot{V}(t)$ peaks with 54% of the recoverable oil extracted. Although the peak is near the midpoint in production, $\dot{V}(t)$ is slightly skewed in both cases, and growth in the production rate is sustained somewhat beyond the midpoint. By using a different approach than ours involving a statistical analysis of the process of discovering finite resources, Bardi[14] also found slight delays beyond the midpoint for a wide range of parameters in his model of oil production.

The asymmetry in $\dot{V}(t)$ means that there is more rapid decline in the production rate after the peak. This asymmetric decline can be made more severe by drilling strategies that attempt to increase the production rate near its peak. Figure 1(c) shows a similar Gaussian curve for $A(t)$ as in Fig. 1(b) until the time $\dot{V}(t)$ peaks, after which $A(t)$ is immediately doubled. The effect is an immediate increase in $\dot{V}(t)$. But there is no delay in the timing of the peak, and the cost of this increase is a precipitous drop after peaking because the total recoverable oil is unchanged.

In real oil fields the number of active wells to which the cross-sectional area is roughly proportional increases as production ramps up and typically reaches a plateau as a field matures. However, the economics of diminishing returns dictates that the number of active wells must eventually decrease to zero as production tails off and wells with low production are shut down. Thus a singly peaked curve is a realistic choice for $A(t)$.

One important implication of the model is that the production rate $\dot{V}(t)$ always peaks before any bell-shaped $A(t)$ peaks. Figure 1(b) shows an example of $\dot{V}(t)$ peaking before the maximum in $A(t)$. This counterintuitive result is consistent with historical data (see Fig. 2) and follows from the requirement, with or without reinjection, that during production $\dot{V}=Af$, where $f=f(V)$ is a positive function with units of velocity and $A(t)$, $V(t)$, and $\dot{V}(t)$ are also intrinsically positive. For our model and the case of secondary recovery, we assume

$$f(V) = x_0\sqrt{\frac{2P_{\rm wh}}{\rho}}\left(1-\frac{V}{V_{\rm rec}}\right). \tag{5}$$

For the case of primary recovery, the velocity of oil exiting a well pipe monotonically decreases over time as oil is produced and downhole pressure drops. In either case, we expect that $\dot{V}(t)$ initially increases with increasing $A(t)$ but ultimately goes to zero for any oil field. Thus any reasonable functional form of $f(V)$ must decrease as the total volume extracted increases, implying that $df/dV<0$. The peak in the rate of production is given by $\ddot{V}=\dot{A}f+A\dot{V}df/dV=0$. Thus, at the peak $\dot{A}=-A\dot{V}f^{-1}df/dV$, which implies that $\dot{A}>0$. Thus, $A(t)$ cannot peak until after the peak of $\dot{V}(t)$. We conclude that based on simple physical considerations, our model predicts that even as the number of active wells is increasing, the production rate will begin decreasing. It can be difficult to anticipate peak production from noisy real-world data, but the model shows that drilling activity is not a useful indicator because an increase in the number of wells does not signify that production will rise.

Another implication of our model is that it provides a simple physical basis for peak production and the decline-from-the-midpoint behavior typically observed at large oil fields. If $A(t)$ is constant, $\dot{V}(t)$ declines monotonically due to a decrease in the oil fraction. To increase $\dot{V}(t)$, which is required for the development an oil field, $A(t)$ must increase initially. Because the total recoverable oil is finite, $\dot{V}(t)$ must eventually decline to zero regardless of how $A(t)$ is varied. For choices of $A(t)$ that mimic the temporal pattern of drilling at real oil fields, solutions of Eq. (4) result in production rate curves $\dot{V}(t)$ that approximately decline from the midpoint in production.

Alternatively, when $A(t)$ is chosen to be in proportion to the current total volume extracted $V(t)$, Eq. (4) is identical to the logistic differential equation. The derivative of the solution of the logistic differential equation, which declines exactly from the midpoint, is the functional form used by Hubbert to fit U.S. oil production data and thereby extrapolate peak production for the lower 48 states.[1,2,6]

IV. TESTING THE MODEL

To compare the model to actual oil production, data were collected from issues of the *Oil & Gas Journal* dating back to 1973.[16] The dependability of the data varies greatly with the source country and company. Besides misreported data, some companies and some countries chose to report detailed data only sporadically or for brief time intervals. However, due to the large number of oil fields reported, useful information can still be recovered.

We fit Gaussians using least-squares regression to data for the number of active wells in the most complete data sets, assuming an average production tubing diameter of 3 in. We chose a Gaussian functional form because we wanted a simple generic bell-shaped curve with few free parameters to represent the function $A(t)$. Wellhead pressure P_{wh} and the initial oil fraction x_0 were given generic values, and the volume of total recoverable oil V_{rec} was estimated by summing past production. Equation (4) was then solved numerically using these three parameters and the Gaussian fit for $A(t)$ for each oil field. The model's prediction for the rate of production $\dot{V}(t)$ can then be compared to data for the actual rate of production at oil fields.

Figure 2 shows data for the number of active wells and the rate of oil production at major oil fields from four continents. In each case the number of active wells is increasing as the production rate peaks and begins to decline, consistent with the model. Figure 2 also shows Gaussian fits to the number of active wells and the model's corresponding result for the production rate $\dot{V}(t)$. For the Forties field (see Fig. 2(a)), all free parameters in the model were calculated directly from additional available data.[17] For the other fields the parameters x_0 and V_{rec} were varied within a range of reasonable values to improve the match between model predictions and data (the final values of the parameters used for each of the data sets in Fig. 2 are given in the caption). The data sets displayed were selected for their geographical distribution and for the quantity and apparent quality of the initial data, not for the quality of agreement between the model and data for the rate of production. Yet, as Fig. 2 shows, we found surprisingly good agreement between the model and data for the rate of production in both the timing and magnitude of the peak.

V. CONCLUSIONS

Our model includes several drastic idealizations of real-world behavior. We assume an unrealistically symmetric $A(t)$ when we fit the active well data to a Gaussian curve, which implies a nearly symmetric curve for the rate of production. A more realistic fit to data for $A(t)$ with sudden onset at the time the first well opens would lead to the more strongly asymmetric rate of production observed in the data. Our model neglects a host of site-specific details such as topography, subsurface geology, and the spatial distribution of wells. We did not attempt to carry calculations beyond a single significant figure or fine-tune the model to achieve better results because the uncertainty in even the best data severely limits quantitative predictions. Furthermore, many of our initial assumptions (well-mixed downhole fluids, no downhole pressure drop, and singly peaked $A(t)$) are only approximations.

Despite these idealizations and our simple approach, our model of oil recovery matches historical data remarkably well and provides valuable insight into key physical processes determining peak production. The model provides a straightforward explanation based on simple physical considerations of decline-from-the-midpoint behavior in large oil fields dominated by secondary recovery. This explanation sheds light on why Hubbert's approach to predicting peak oil was successful.

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APPENDIX: SUGGESTIONS FOR FURTHER STUDY

(1) Fluid exit velocity.

(a) Derive Eq. (2) without starting from Bernoulli's principle. Consider the forces acting on a uniform column of incompressible fluid open at the top but under pressure at the bottom.

(b) How high would you expect an oil gusher to reach if the wellhead cracked? How does it compare to the world's tallest fountains? What unmodeled effects might limit the height of a gusher?

(2) Use a software package or write a program to integrate Eq. (4) numerically given $A(t)$ and reproduce Fig. 1. For

Fig. 1(a) use the logistic function $A(t)=A_{max}A_0e^{rt}/[A_{max}+A_0(e^{rt}-1)]$. For Fig. 1(b) use $A(t)=A_{max}\exp[-(t-t_{max})^2/(2\sigma^2)]$. Try both realistic parameters (as given in the caption of Fig. 2) and arbitrary parameters.

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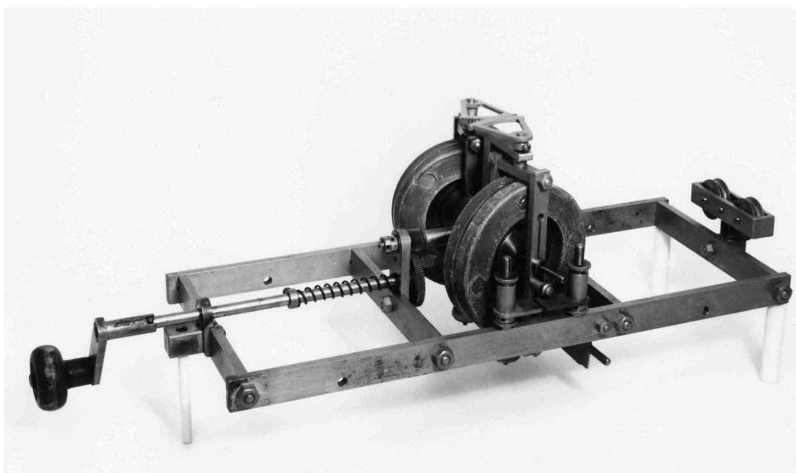

Brennans Monorail Car. The early years of the twentieth century saw a good deal of interest in monorail railroads. This model of the monorail car designed by Louis Brennan was originally described in Nature in 1908, and subsequently sold by the Central Scientific Co. at a price of $150. This version was designed by Henry Crew and Robert Tatnall of Northwestern University. The device, with its contra-rotating gyroscopes, runs on a taut wire cable. The overall length is about 26 inches. This example came to the Smithsonian Institution from Columbia University in 1963, and is accession No. 249,200. I would be interested in hearing from anyone else who has a example of this instrument. (Smithsonian photograph #72-1306; notes by Thomas B. Greenslade, Jr., Kenyon College)